Stellar Angular Diameters

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History of Angular Diameter Measurement

- Michelson & Pease
- Narrabri - intensity interferometry
- Lunar occultations
- Amplitude interferometry
  - Mark III, IOTA
  - NPOI, PTI

Basic Parameters from Angular Diameters (q)

- Effective temperature is defined as: $L = 4p\,s\,R^2 T_{\mathrm{EFF}}{}^4$,

  which can be rewritten as: $T_{\mathrm{EFF}} = 1.316\times10^7 \left(\frac{F_{\mathrm{BOL}}}{q_{\mathrm{R}}^2} \right)^{1/4}$

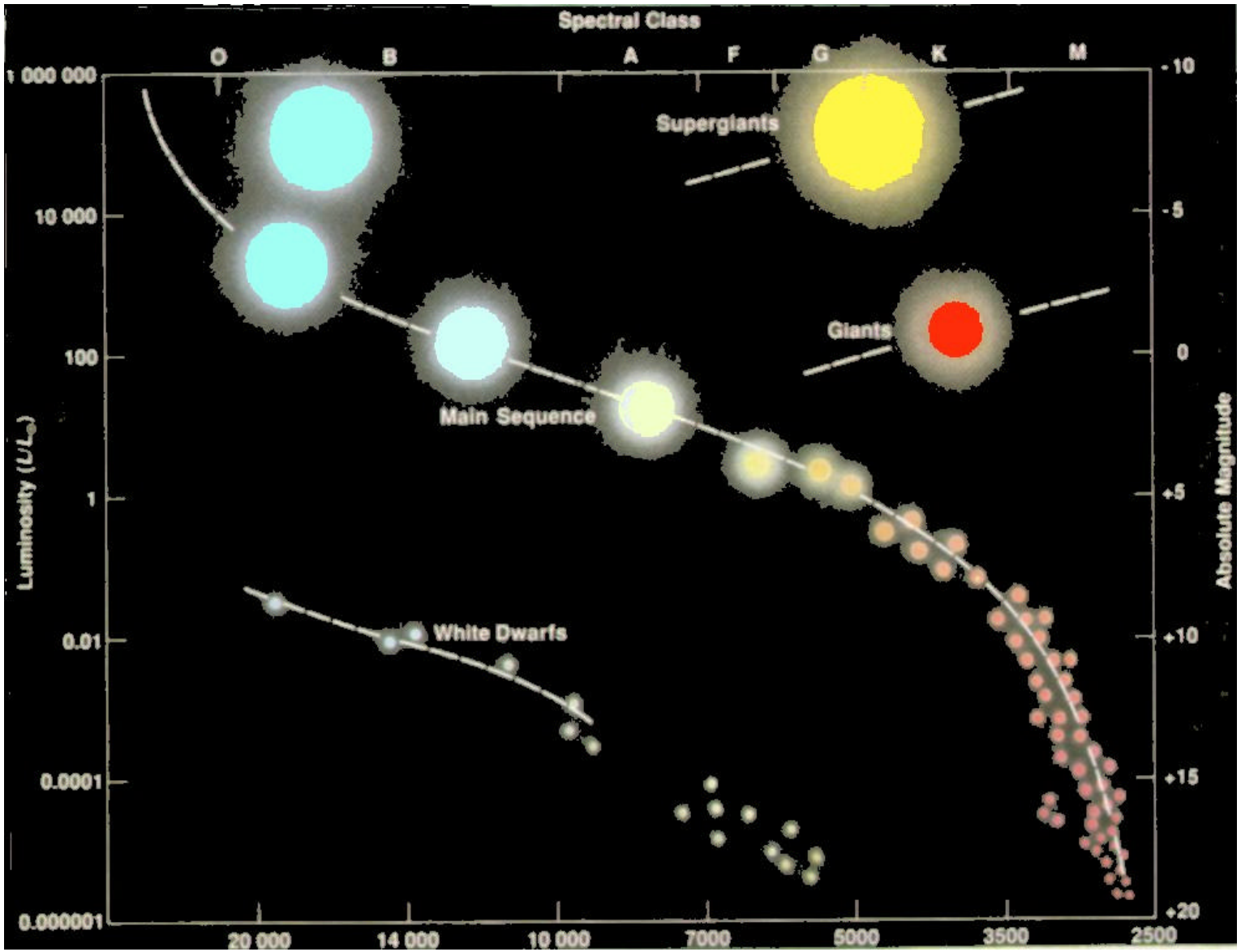
  – F_{BOL} is the bolometric flux (W cm^{-2}), q_{R} is the Rosseland mean stellar angular diameter (mas)

- Linear radius is simply: $R = \frac{1}{2} q \times d$

  – Hipparcos (Perryman et al. 1997) distances now available

  – Uncertainties in parallax (typically ~15-20%) still largest contribution to error

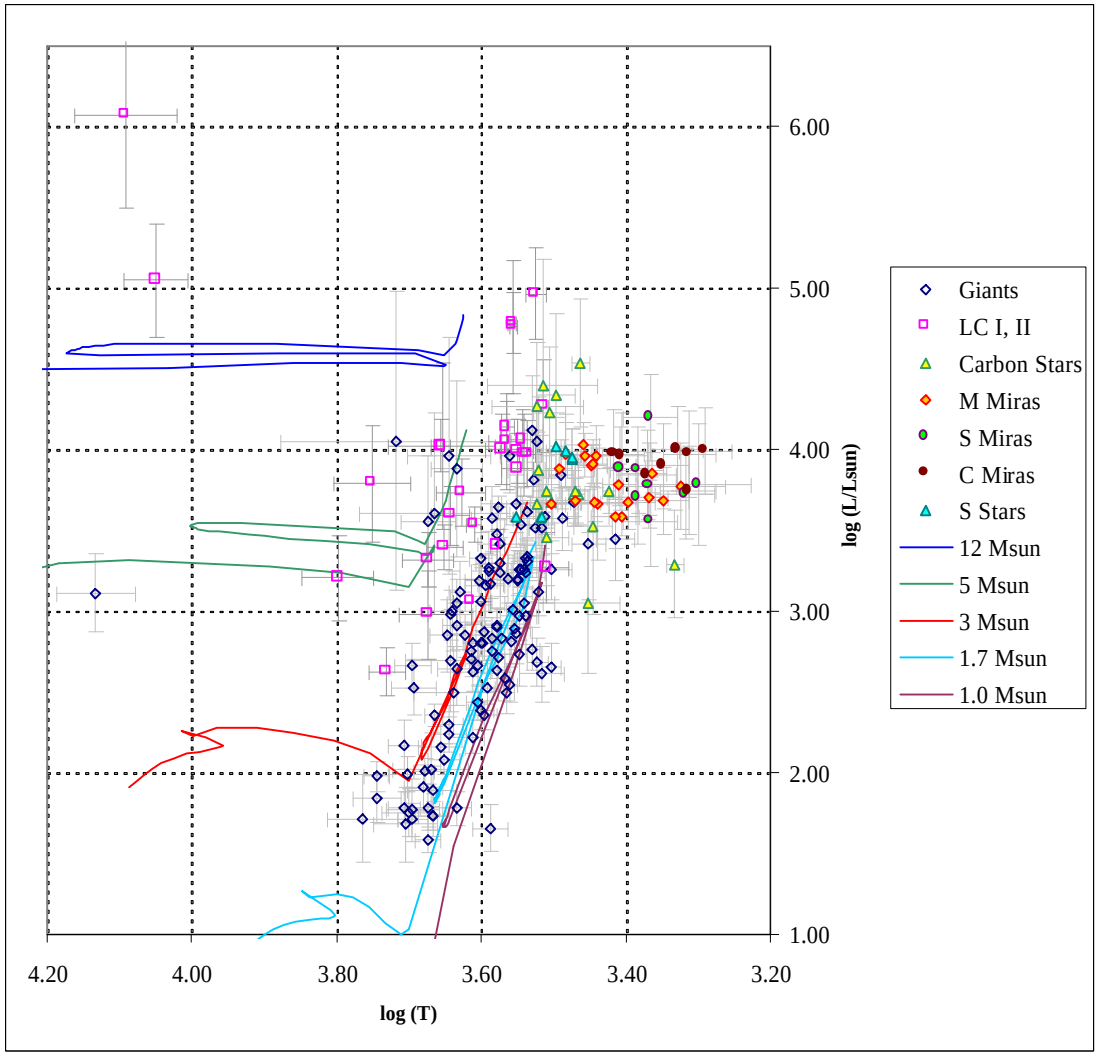
HR Diagram. I


Spectral Class
O
B
A
F
G
K
M
Supergiants
Giants
Main Sequence
White Dwarfs
Luminosity ($L/L_\odot$)
1 000 000
10 000
100
1
0.01
0.0001
0.000001
Absolute Magnitude
-10
-5
0
+5
+10
+15
+20
20 000
14 000
10 000
7000
5000
3500
2500

HR Diagram. II

- Key tool in understanding life cycles of star
- Model Tracks:
  - All tracks are Z=0.02 (solar)
  - Tracks from 1.0 - 12 solar masses

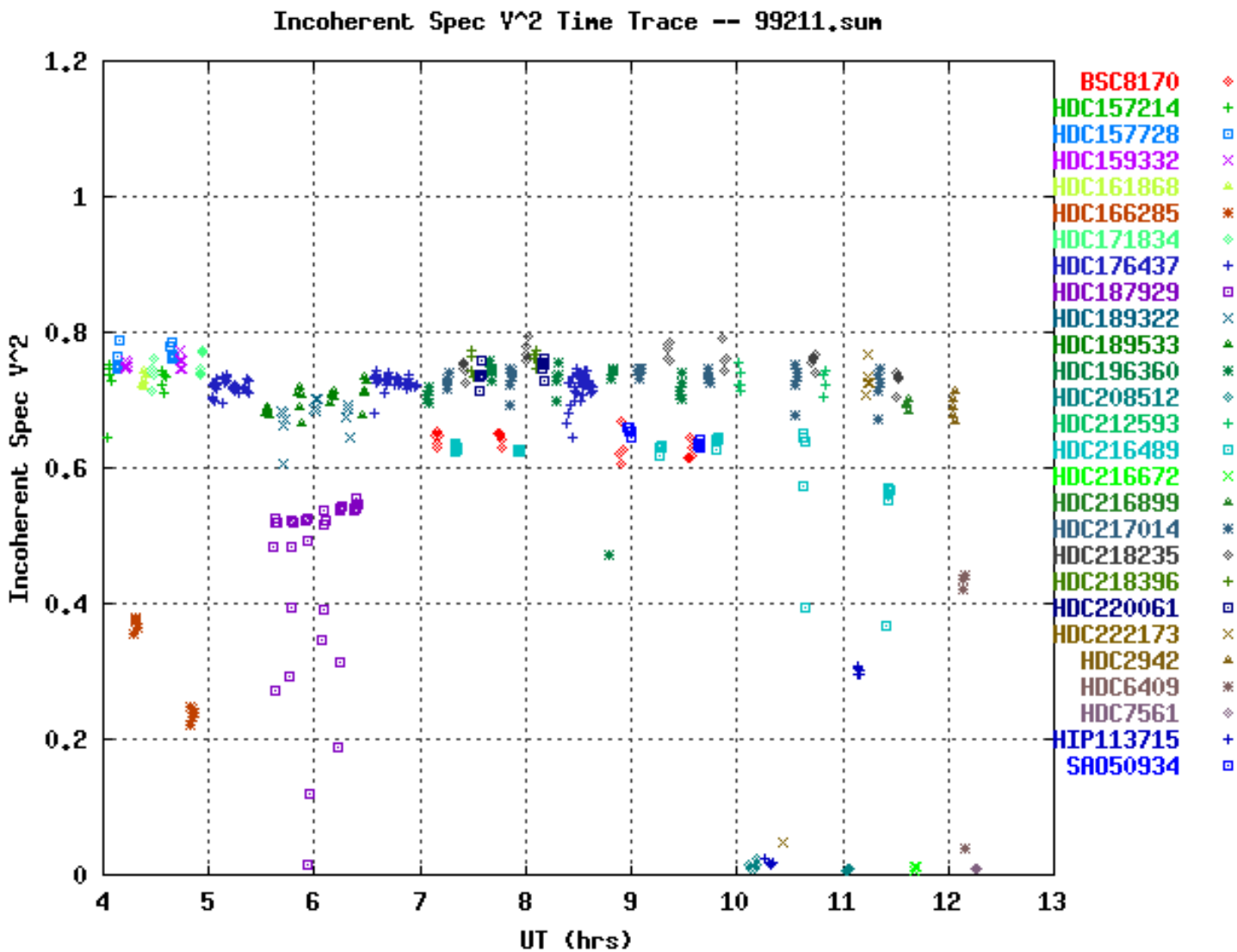
    (Schaller *et al.* 1992, Charbonnel *et al.* 1996)



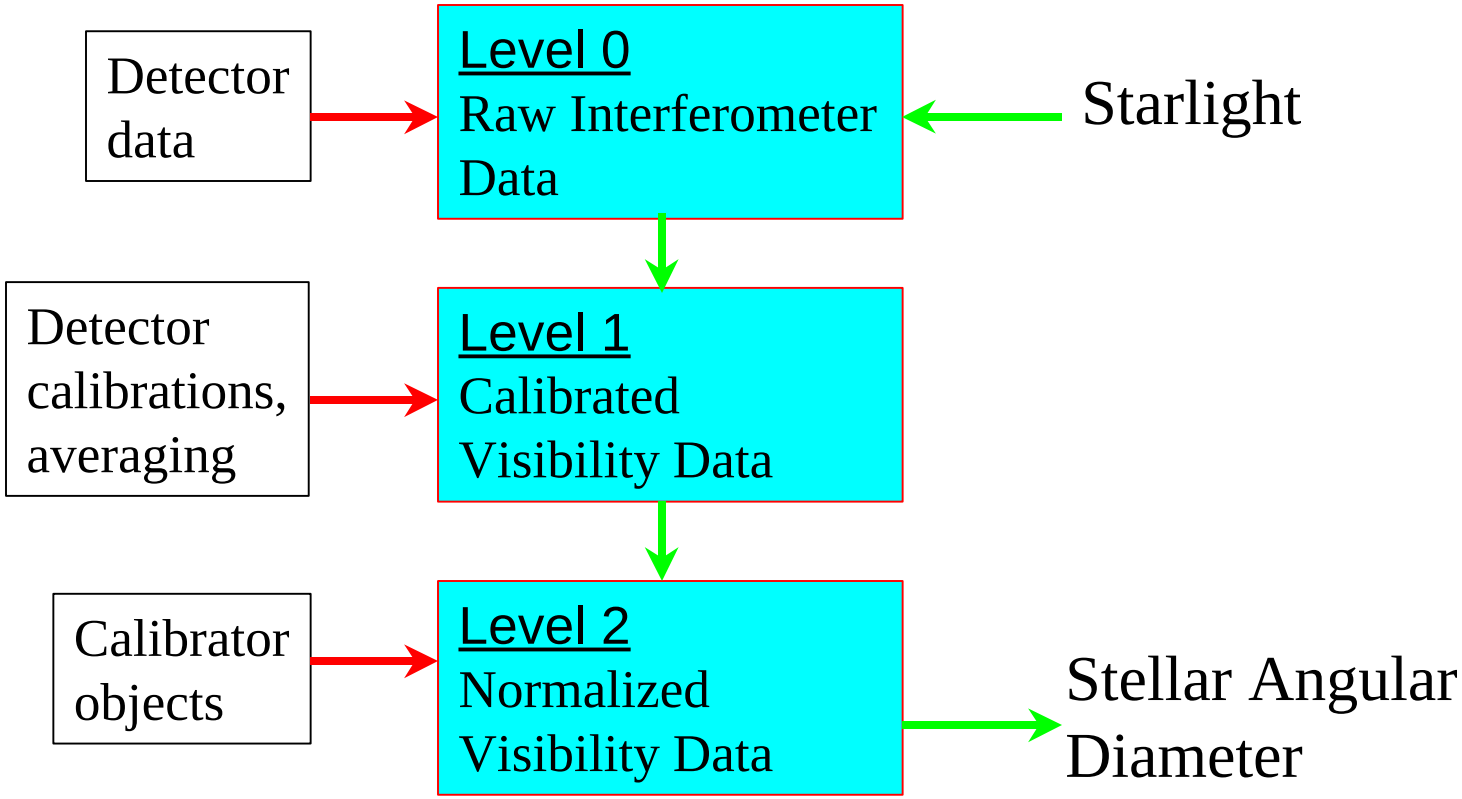
Ancillary Data

- Visibility data largely impotent without it
- Photometry
  - Contemporaneous data important for variable stars
- Distances
  - Note errors in size vs. errors parallax problem
- Spectroscopy
- Radial velocity data
- A well-thought out observing program collects interferometric and ancillary data!

Visibility Data from an Interferometer



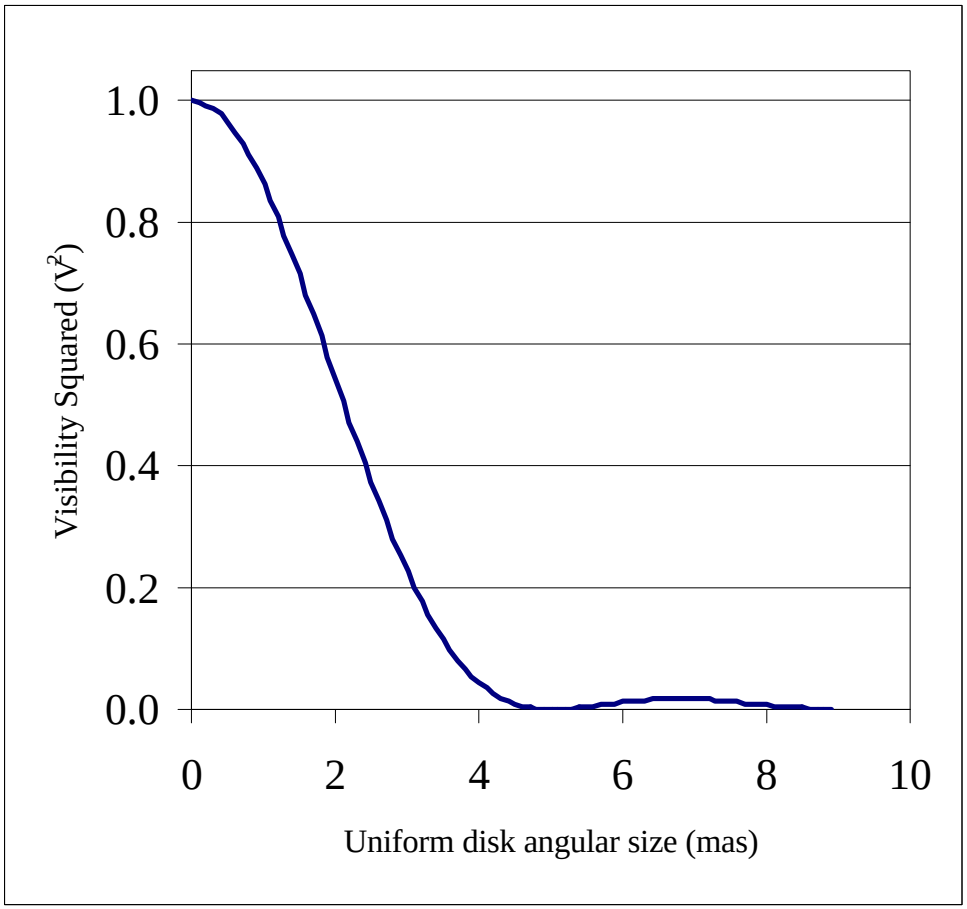
Visibility Data Reduction



Uniform Disk Diameters from Visibilities

- Solve for uniform disk diameter q_{UD}
  - Use $V^2 = (2J_1(x)/x)^2$ in central lobe;

    $x = \pi q_{UD} B_p / l$
- Assumes source is a top hat
- Work in central lobe of visibility function - nondegenerate



Current Stock of Results

- Borrowing from Davis (1997), increase of 145 to 340 stars in the literature
  - Largely due to sizes published by Dyck & van Belle
  - Noting that 78 of the original 145 are still unpublished
- Notable improvement: Application of interferometry to evolved stars
- Notable area for improvement: *Still* main sequence stars, particularly late-type

Spectral Type	I	II	III	IV	V
O	3	0	0	0	1
B0-B4	2	2	3	2	2
B5-B8	2	0	2	1	1
A0-A3	1	0	0	2	5
A5-A7	0	0	1	0	1
F0-F5	4	1	0	1	0
F8	2	0	0	0	0
G0-G5	3	1	2	3	0
G7-G9.5	2	1	22	0	0
K0-K3.5	5	16	31	0	0
K4-K7	3	1	14	0	0
M0-M4	12	13	70	0	0
M5-M8	1	2	31	0	0
Totals	40	37	176	9	10

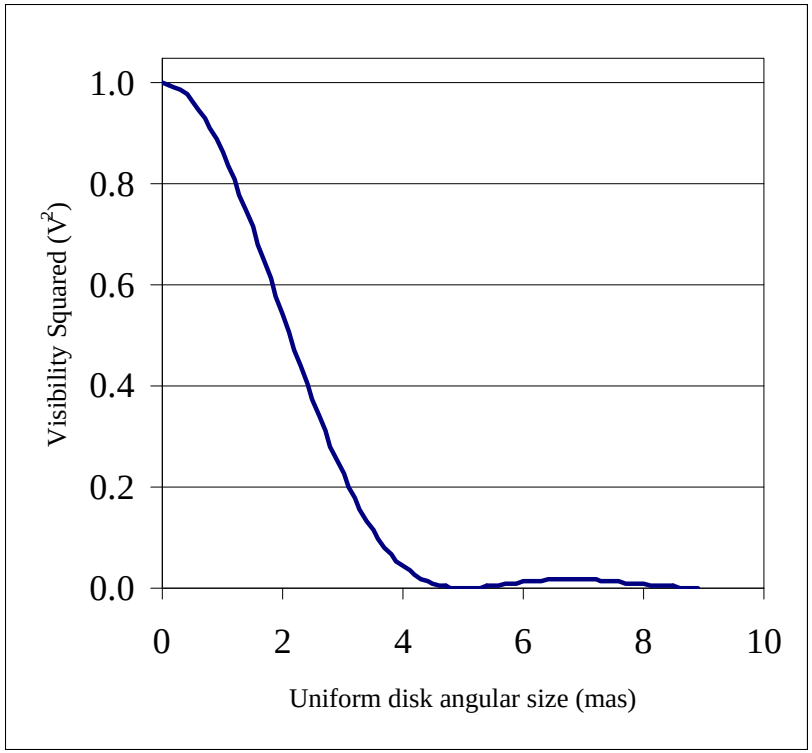
Evolved Stars	
Carbon	22
M Miras	37
C Miras	5
S Miras	4
Total	68

Conventions and Caveats

- Angular *diameters* typically given, in contrast to linear *radii* (watch that factor of 2!)
- Luminosity is not defined by q
  - Luminosity goes as $L \sim R^2T^4 \sim d^2F_{BOL}$
  - Limits HR diagrams

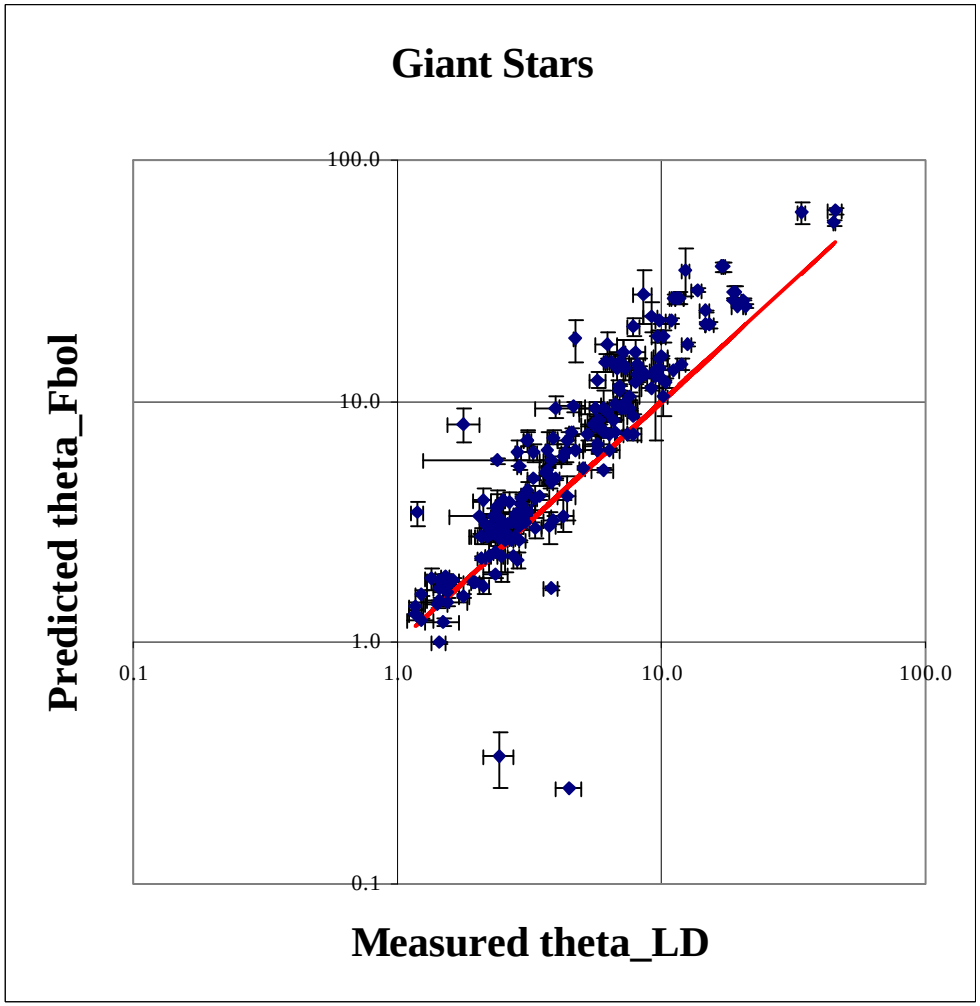
Calibrator Objects

- Need to establish instrument point-source response
- Chicken & the egg
  - Need to predict sizes to measure sizes
  - Key to unraveling problem is using nonlinearity of visibility curve
  - Calibration issues push sensitivity limits of instrument



Prediction of Stellar Angular Sizes. I.

- Linear radius - distance
  - Radius vs. V-K, spectral type
  - Distances hard to come by
- BBR assumptions
  - Systematic differences

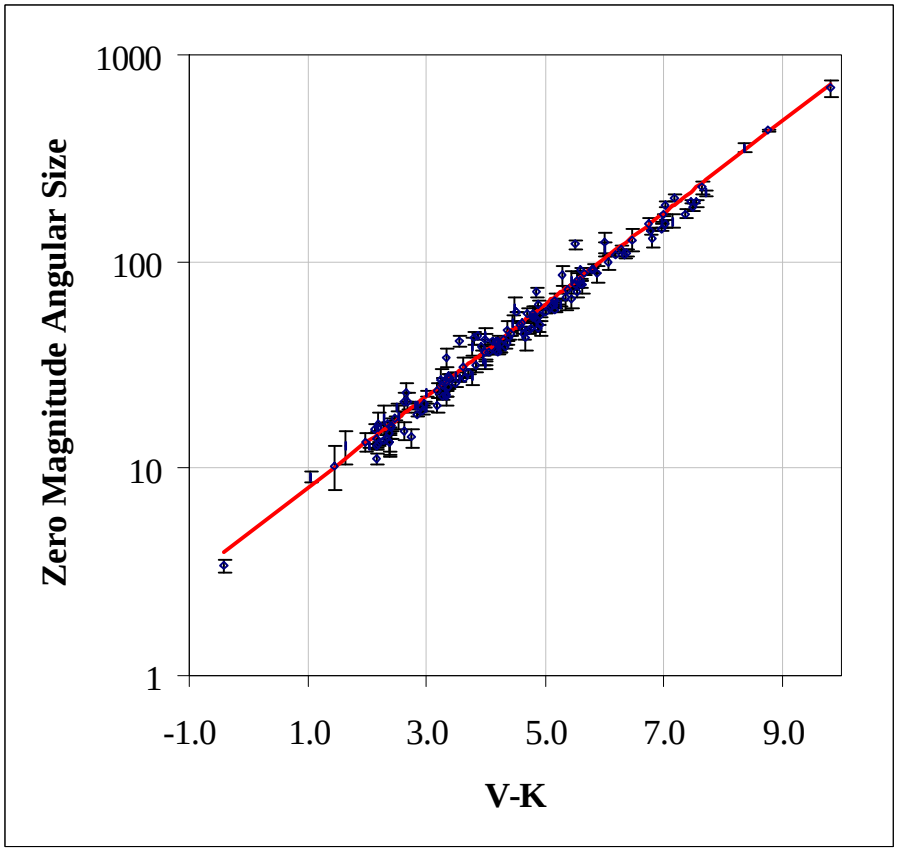


Prediction of Stellar Angular Sizes. II

- Define a zero magnitude angular size

$$q_{V=0} = q \times 10^{V/5}$$

- Unaffected by interstellar reddening
- Uses only V, K magnitudes; accurate to 11-12% for LC I-III
  - Also useful for LC V, more evolved sources
- Error sources
  - Angular size errors
  - V-K errors
  - Unparameterized phenomena - 'natural dispersion'
- Recently accepted for publication (van Belle 1999)

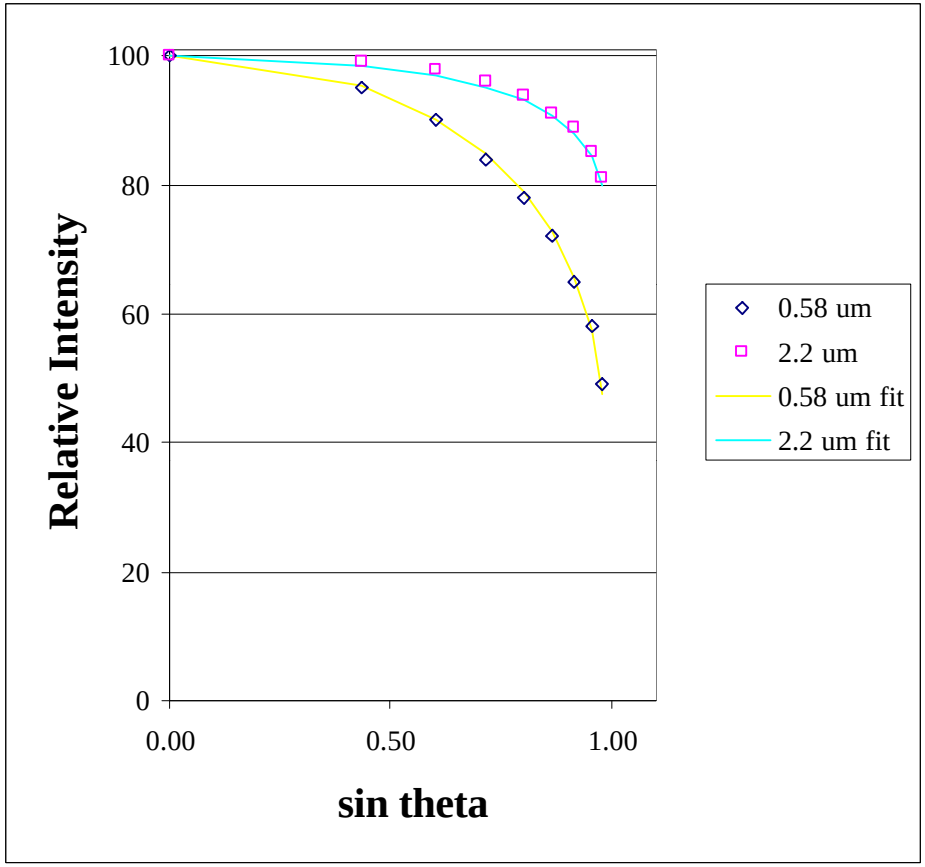


Uniform Disk vs. Limb-Darkened Diameters

- Solar values: Pierce & Waddell (1961)
- Fit law

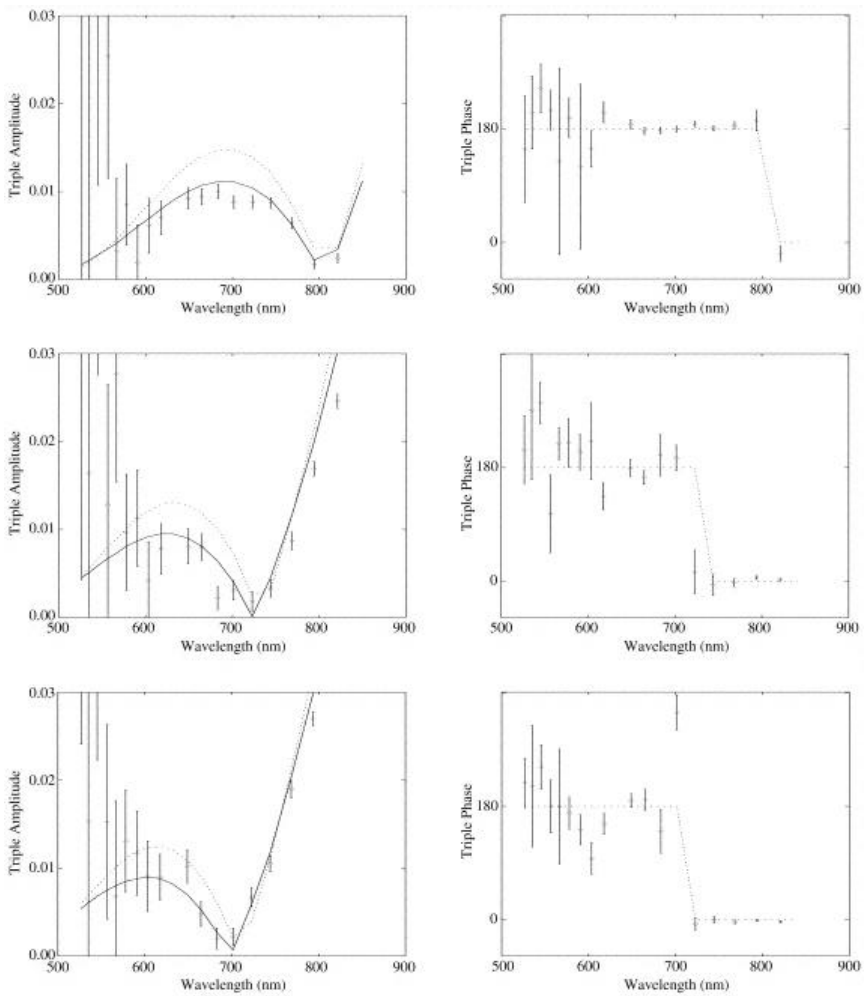
$$I(q)/I(0) = (1-\sin^2 q)^n$$

- n(0.58um)=0.23, n(2.2um)=0.07



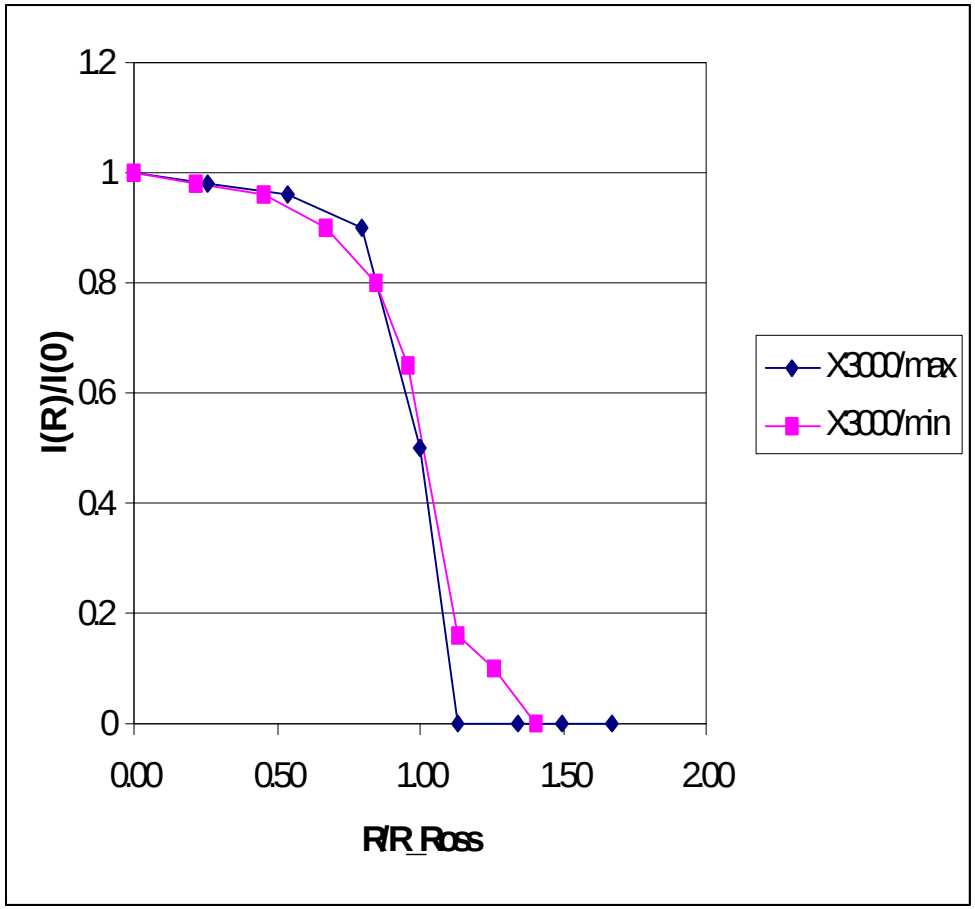
Uniform Disk vs. Limb-Darkened Diameters

- See Hajian et al. (1998)
- Size of α Ari (a K2III giant):
  - θ_{UD} = 6.34+-0.06
  - θ_{LD} = 6.80+-0.07



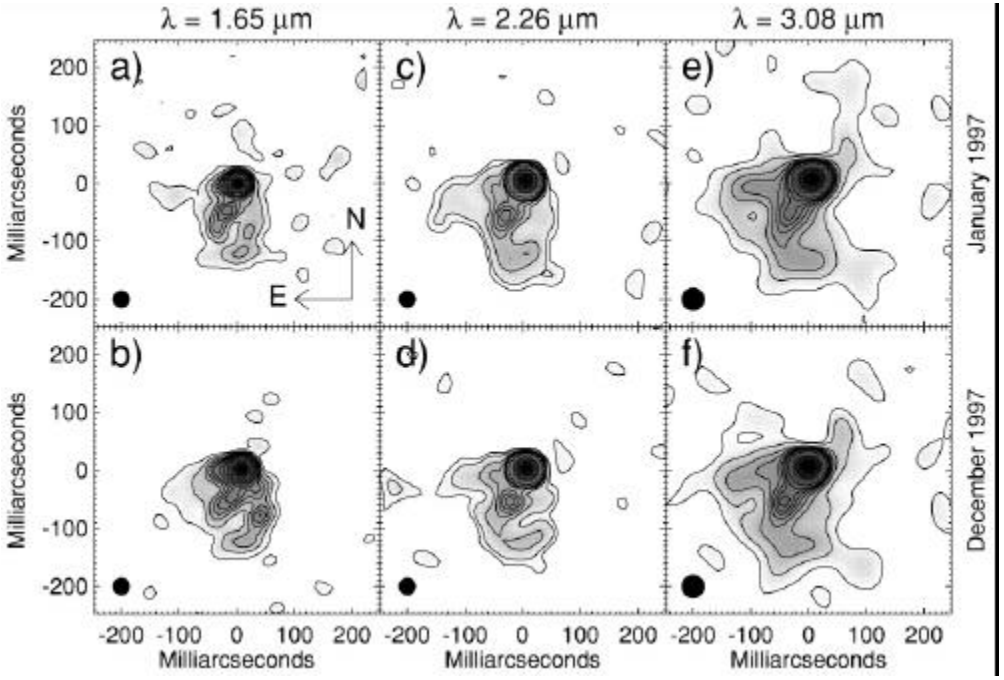
Model Center-to-Limb Predictions

- Good example: Scholz & Takeda 1987
  - Can compare θ_{UD} to θ_{ST}
  - 3000K max light model: 0.99
  - 3000K min light model: 1.17
- Very separate effects for 3500K models, Z=nonsolar, etc.



Higher Order Surface Structure

- Hot/cool spots
- Nonsphericity
  - Evidence from Monnier et al. (1999) and Tuthill et al. (1999) of asymmetries in stellar envelopes



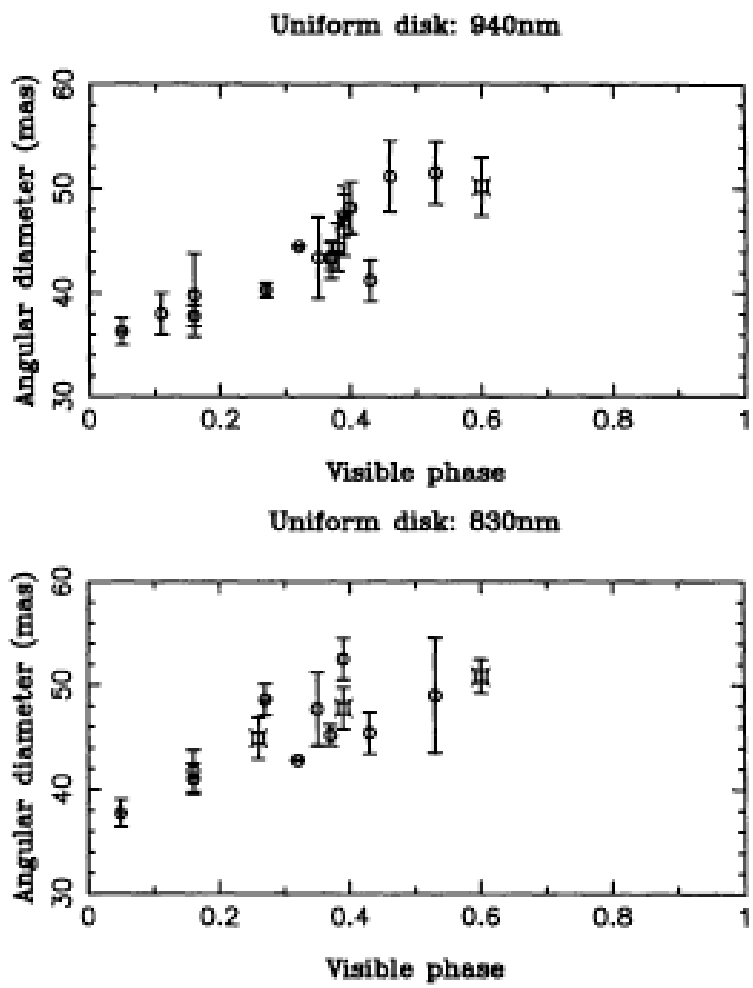
VY CMa
M5eIbp
$L \sim 2 \times 10^5\ L_{SUN}$

Wavelength Dependent Effects

- Molecular absorption
  - K band: ^{12}CO bandhead at 2.29μm
  - Red: TiO at 710 nm
  - Good references for the visible band effects are MkIII papers and COAST papers (eg. various papers by Haniff et al.)
- Mean radiating surface is difficult to extract from angular sizes strongly affected by absorption effects
- Actually examining different layers of the atmosphere

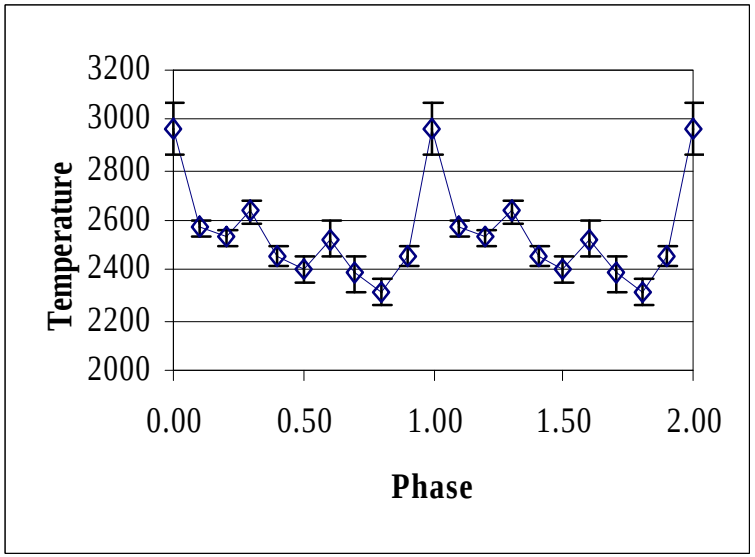
Temporal Effects

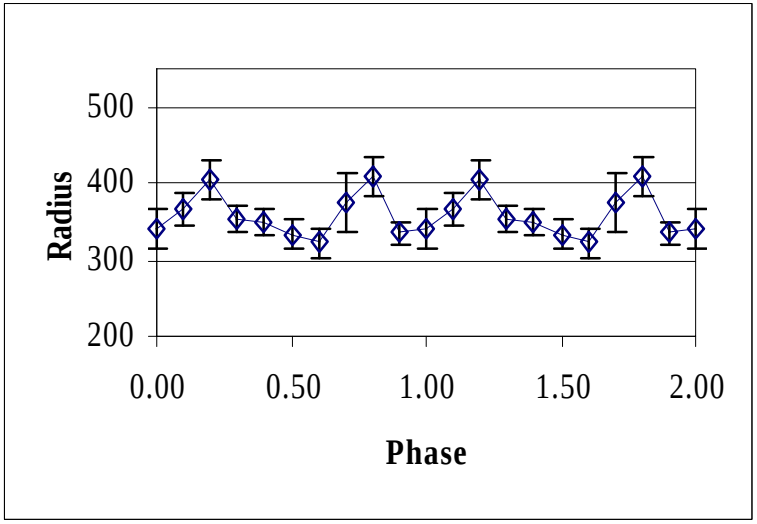
- Progression of T, R with phase for variable stars
  - Mira variables
  - Cepheid variables
  - Semi-regulars
- Best single star results: R Leo at COAST (see Haniff et al. 1998, at right)
- Previous, forthcoming IOTA results (see van Belle et al. 1996, 1997; Milan-Gabet et al. 2000?)



Mira Variables: *T*, *R* Versus Phase

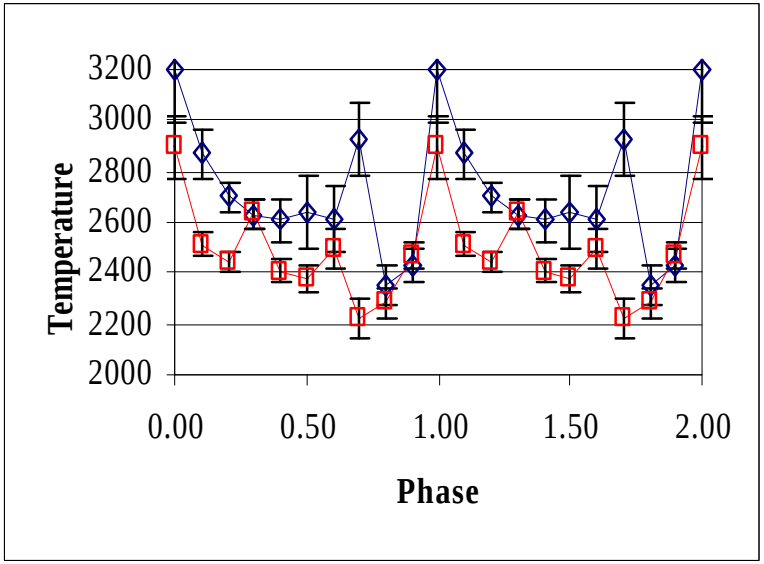
- Zero-phase corresponds to maximum light as determined from V band photometry
  - Maximum *T*, *R* minima
  - Also thought to correspond to the star expanding through its equilibrium radius with maximum velocity (Kanbur et al. 1997)
- Second *R* minima seen at ϕ~0.6 - possible explainations:
  - Dissimilar Miras in sample set
  - Observation of separate atmospheric layers (eg. shells?) and/or shocks

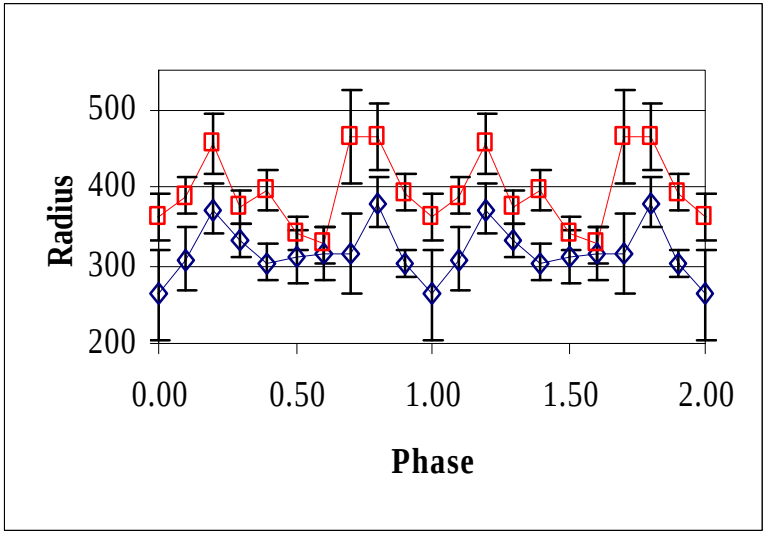




T, *R* vs. Phase: Short vs. Long *P* Miras

- Data set separated into 'short' (*P*<350d, blue) and 'long' (*P*>350d, red) period Miras
- Consistent temperature, radius differences
  - $T_{350+} / T_{350-} = 0.93 \pm 0.01$
  - $R_{350+} / R_{350-} = 1.20 \pm 0.05$
- Indicates $L_{350+} / L_{350-} = 1.07$
  - Consistent with longer-period Miras being more evolved, more luminous objects
- Double-peaked structure still evident in *R* vs. ϕ





Pitfalls in the Esoteric Art

- Binarity
- Combination of resolution / sensitivity / calibration
  - Rule of thumb: You want no more than 100:1 ratio of baseline to aperture
    - Example: PTI's 110m baseline /0.4m apertures, K<4.5 limit
  - Multiple wavelength needs of an instrument tend to exclude 'the interesting stuff'
- Treat your instrument with the utmost suspicion
  - See something interesting? Probably an instrumental artifact

Future Directions

- Main sequence objects
  - Larger apertures, baselines: SUSI, CHARA, NPOI
- Easily extended to non-stellar objects
- Pushing on stellar atmospheres models
  - Wavelength dependencies of angular sizes
  - Characterization of higher order surface structure