Amplitude-sensitivity of phase detection in the metrology gauge

RES

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1 Experiment description

This memo describes a measurement of the amplitude sensitivity of the laser board phase meter. Ideally, the phase meter does not respond to variations of the amplitude of the input;[1] in practice, imperfections in the analog electronics can cause intensity fluctuations, as might be inherent in the laser source, to appear as false changes in optical path. The experimental setup is described in the logbook entry for 25 August 1998. Light emitting diodes were substituted for the optical input from an interferometer. Phase modulation was used to check the calibration of the phase meter, and amplitude modulation to measure the amplitude-sensitivity.

The light intensity of one LED ("reference") was modulated at the nominal heterodyne beat frequency of $\omega = 100\,\mathrm{kHz}$; the other ("unknown") LED had the same signal plus phase or amplitude sinusoidal modulation of frequency $\Omega = 78$ Hz. The LED instantaneous and peak intensities are designated V and V_0, respectively.

Phase modulation with "modulation index" Γ_p generates sidebands spaced at multiples of Ω on either side of ω. The magnitude of the first sidebands is $\Gamma_p/2$:

$$V/V_0 = \cos(\omega t + \Gamma_p \cos\Omega t) \tag{1}$$

$$\approx \cos\omega t - \frac{\Gamma_p}{2}\left[(\sin([\omega+\Omega]t) + \sin(\omega-\Omega]t)\right]. \tag{2}$$

The approximation is valid for $\Gamma_p << 1$. Amplitude modulation of magnitude Γ_a generates sidebands first sidebands that look similar:

$$V/V_0 = \cos\left(\omega t(1+\Gamma_a \cos\Omega t)\right) \tag{3}$$

$$= \cos\omega t + \frac{\Gamma_a}{2}\left[(\sin([\omega+\Omega]t) - \sin(\omega-\Omega]t)\right]. \tag{4}$$

2 Results

The output of the phase meter is in "cycles" of phase, where one cycle corresponds to $\lambda/2$ displacement. The calibration was verified by setting up the signal generator to generate a phase-modulated sinusoid, with Ω sidebands 26 dB (a factor of 20) below the "carrier" at ω. According to Equation 2, this corresponds to $\Gamma_p = 0.1$. The phase meter read out a sinusoid of amplitude 0.0159 cycles 0-p, equal to $\Gamma_p/(2\pi)$, as expected.

The results for amplitude modulation are shown in Table 1. Although the measurement was made using sinusoidal amplitude modulation, it is expected that the results apply to all frequencies down to and including d.c.

3 Effect of comparator offset

An offset in the comparator circuit that converts the heterodyne sine wave to a square wave would cause amplitude sensitivity. If the offset is V_1, then the comparator triggers not at $\phi = 0$, but rather at ϕ_1 such that

$$V_1 = V_0 \sin\phi_1, \tag{5}$$

[1] We consider here only fluctuations that are slow compared to the heterodyne beat frequency. Arbitrarily fast fluctuations, such as from photon shot noise, result in an unavoidable phase noise—see the article "Heterodyne interferometer noise" at http://huey.jpl.nasa.gov/∼respero.

Γ_a	$\Gamma_a/(2\pi)$	Δn (cycles)	$\frac{\Delta n}{\Gamma_a/(2\pi)}$	$\Delta x = \frac{\lambda}{2}\Delta n$
0.1	0.0159	$8 \cdot 10^{-4}$	0.05	520 pm
0.2			0.035	

Table 1: Phase meter readings (Δn) for various levels of amplitude change (Γ_a.) The Δn column shows the phase meter readout, as provided by the VX-Works Stethoscope front end. The entries in the next column would be unity for phase modulation Γ_p; hence they indicate the dimensionless amplitude-to-phase feedthrough. The Δx column represents the error in mirror position attributable to amplitude change. It is based on $\lambda/2$ rather than λ because the optical path is twice the physical path in a single-bounce interferometer.

or $\phi_1 \approx V_1/V_0$. Differentiating, with $V_1 << V_0$,

$$\Delta\phi_1 = -\frac{V_1}{V_0^2}\Delta V_0 \tag{6}$$

$$= -\frac{V_1}{V_0}\Gamma_a \tag{7}$$

where Equation 7 follows from the definition of the modulation depth, $\Gamma_a = \Delta V_0/V_0$. In terms of displacement error,

$$\Delta x = \frac{\lambda}{4\pi}\frac{V_1}{V_0}\Gamma_a \tag{8}$$

If the Δx in the first row of Table 1 is due to this mechanism, it requires $V_1 = 100\,\mathrm{mV}$, which is implausibly high for a properly wired comparator.

Equation 8 is expected to hold for all frequencies of amplitude modulation, and can therefore be generalized to RPSD's. Designating the RPSD of intensity fluctuations by $\tilde{I}(f)$ and the average intensity by I_{DC}, the RPSD of measured displacement in the presence of an offset is

$$\tilde{x}(f) = \frac{\lambda}{4\pi}\frac{V_1}{V_0}\frac{\tilde{I}(f)}{I_{\mathrm{DC}}} \tag{9}$$

4 Requirements on analog electronics

The largest relevant amplitude change is expected to be over long periods, as from changes in laser output power or gauge alignment. For an allowable intensity feedthrough error of 5 pm and a possible amplitude change as much as 10% ($\Gamma_a = 0.1$), Equation 8 indicates that the offset needs to be no more than 1 mV. Given a spectrum of intensity fluctuations $\tilde{I}(f)$, Equation 9 can be used to estimate the astrometric error.